Class 8 – Relations & Functions

Review

order of operations:

$\sqrt{\ }$ & $a^n \to \times \div \to + -$

→ Evaluate an expression:

$$\frac{3(x+3)}{15x-30}, \quad x=3 \qquad \frac{3(3+3)}{15\cdot 3-30} = \frac{3\cdot 6}{45-30} = \frac{18}{15} = \frac{6}{5}$$

Calculator: $(3\times(3+3))\div(15\times 3-30) = 1.2 \to$ 2nd, PRB $\to$ $6/5$
(2nd A b/c)

→ Solving Linear & Quadratic equations:

$$3(x-4)-15=0 \Rightarrow \frac{\cancel{3}(x-4)}{\cancel{3}} = \frac{15}{3} \Rightarrow x-4=5 \Rightarrow x=9.$$
$\quad +15 \quad +15 \qquad\qquad\qquad\qquad +4 \quad +4$

$$\underset{A}{x^2} + \underset{B}{3x} - \underset{C}{10} = 0 \Rightarrow \left.\begin{array}{l}\text{sum} = 3\\ \text{prod} = -10\end{array}\right\} +5 \ \& -2 \Rightarrow \underbrace{(x+5)}_{a}\underbrace{(x-2)}_{b} = 0$$

$x+5=0 \Rightarrow \underline{\underline{x=-5}}$ or $x-2=0 \Rightarrow \underline{\underline{x=2}}$

→ Simplify expressions:

$$(2x^3-9x^2+7x+5)-(3x^2-6x+8)$$

$$2x^3 - 9x^2 + 7x + 5 - 3x^2 + 6x - 8 = 2x^3 - 12x^2 + 13x - 3.$$

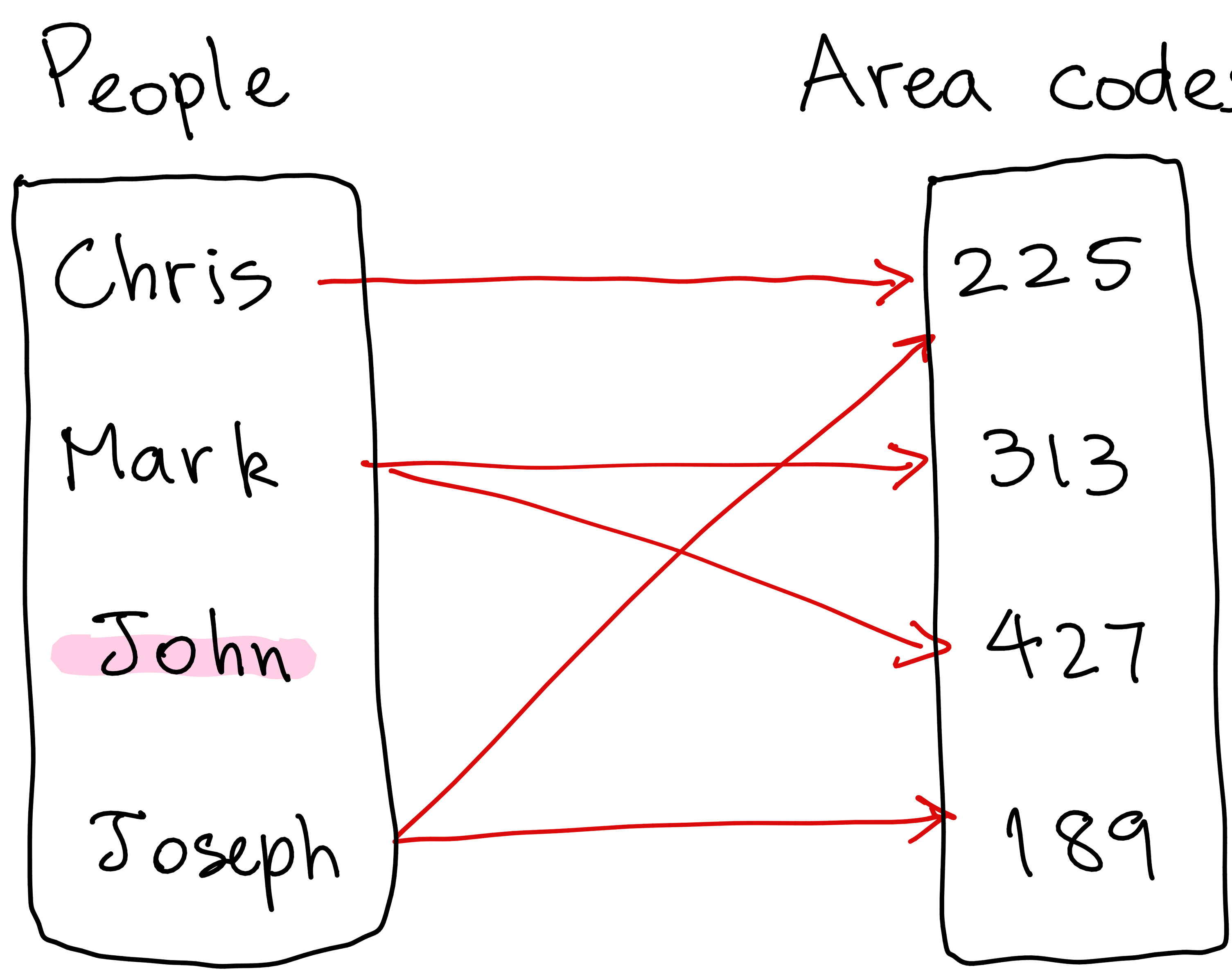
$(a+b)^2 = a^2+2ab+b^2$

$$\left(3-6(x+h)^2\right) - \left(3-6x^2\right)$$

$$= \left(3-6(x^2+2xh+h^2)\right) - \left(3-6x^2\right) = \cancel{3} - \cancel{6x^2} - 12xh - 6h^2 - \cancel{3} + \cancel{6x^2}$$

$$= -6h^2 - 12xh.$$

Relations



$\longleftrightarrow$ {(Chris, 225), (Mark, 313), (Mark, 427), (Joseph, 225), (Joseph, 189)}

Function: each element in the first set is related with exactly one element in the second set.

(every elt. in 1^{st} set has exactly one arrow)

1^{st} set: domain & 2^{nd} set: codomain (range)

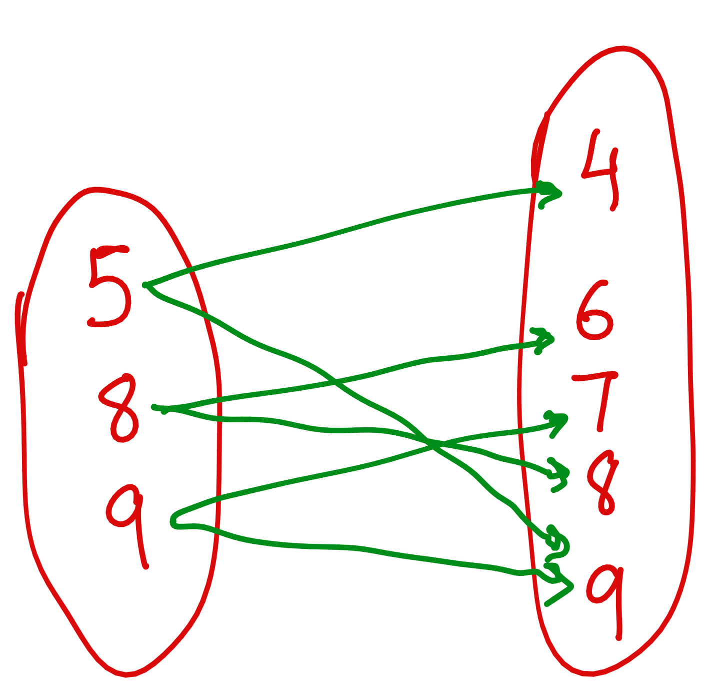
elements in 2^{nd} set that have an arrow

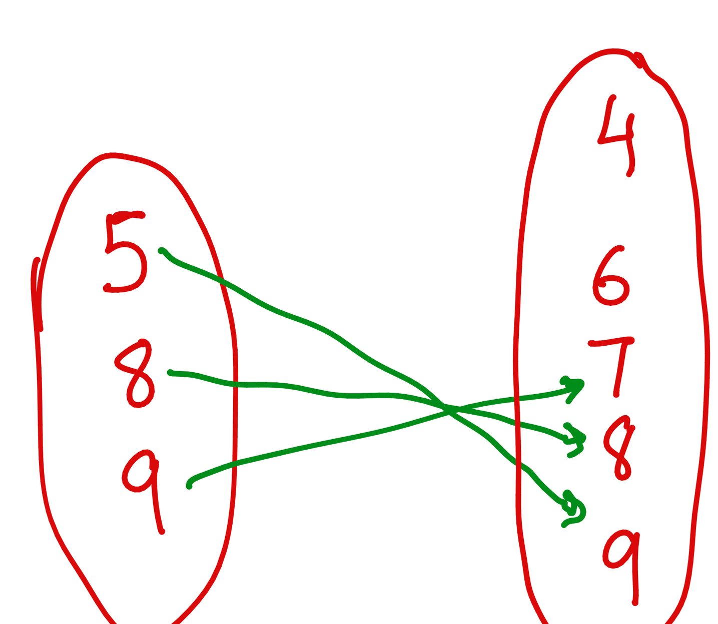
Ex1: Consider the relation $\{(8,6), (5,4), (9,7), (5,9), (8,8), (9,9)\}$

a) What is the domain? $\{5, 8, 9\}$

b) What is the range? $\{4, 6, 7, 8, 9\}$

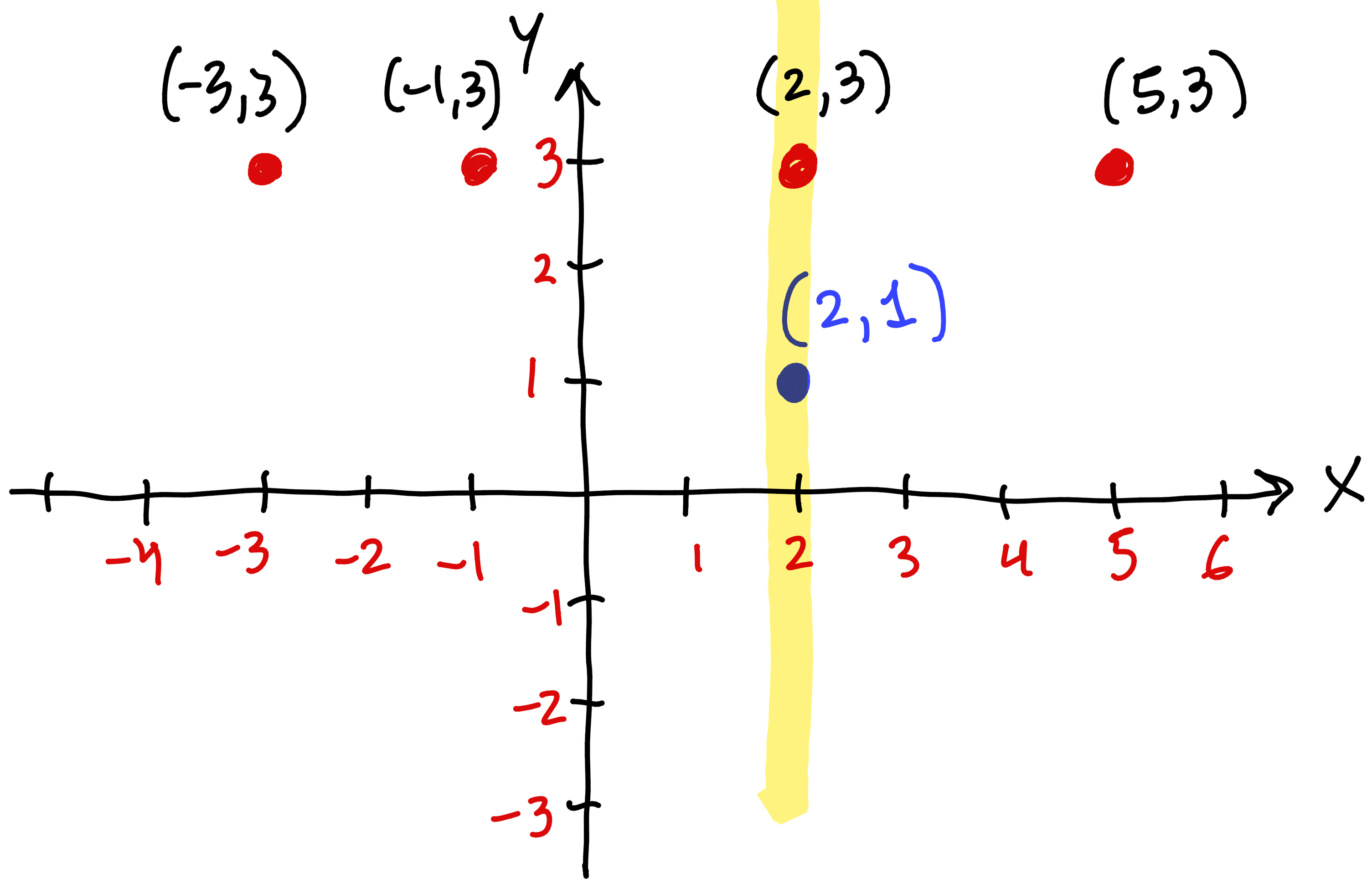
c) Is this a function? No.





This is a function

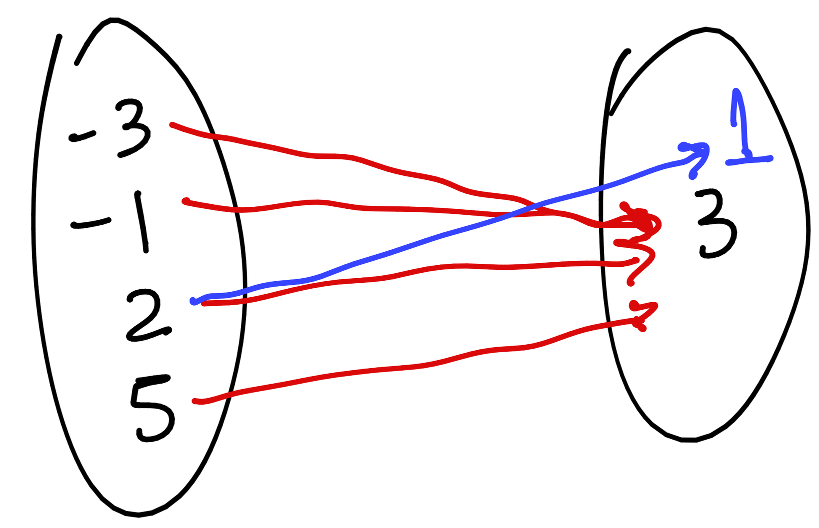
Ex 2: Consider the relation:



a) Domain: $\{-3, -1, 2, 5\}$

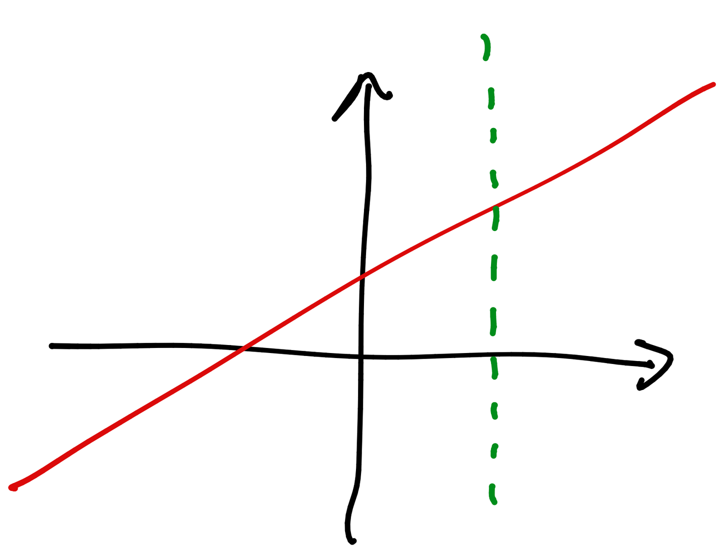
b) Range: $\{3\}$

c) Is a function? Yes

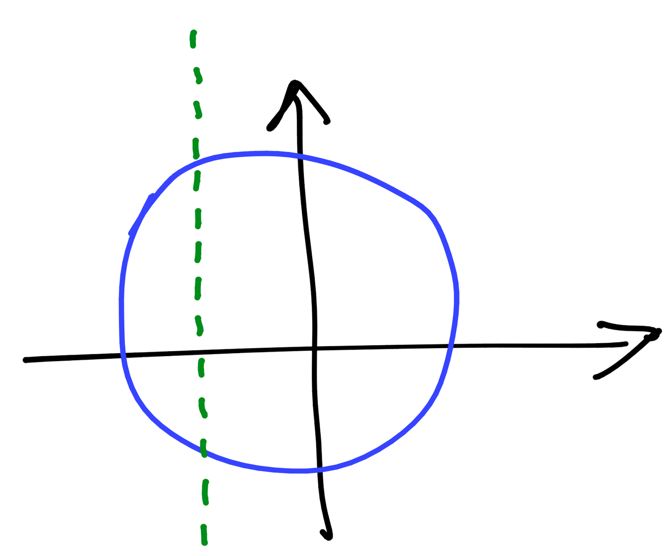


The vertical line test:

NOT a function if some vertical line touches a graph >1 times



function



NOT function

Polynomial: $3x^5 + 2x^{90} - x^{41} + 5 \cdot x^0$ x^n, n is positive integer

Real numbers

$$\frac{2}{x} = 2 \cdot \frac{1}{x} = 2x^{-1}$$

$\rightarrow Ax^2 + Bx + C$ quadratic

$Ax + B$ linear

$$2\sqrt{x} = 2x^{\frac{1}{2}}$$

$f(x) = 2 \cdot x + 1$, $f(0) = 2 \cdot 0 + 1 = 1$, $f(1) = 3$, $f(2025) = 4051$

- the domain of a polynomial function is $(-\infty, \infty)$.

Remark: $g(x) = \frac{2x+5}{x-1}$, $g(2) = \frac{2\cdot 2+5}{2-1} = 9$

$g(x) = \frac{2x+5}{x-1}$ → numerator (polynomial), → denominator (polynomial) → When denom. $= 0$

$x - 1 = 0 \iff x = 1$

Domain: $(-\infty, 1)$ or $(1, \infty)$ $= (-\infty, 1) \cup (1, \infty)$

$(-\infty, 1)$ ↓ all #'s < 1

$(1, \infty)$ ↓ all #'s > 1

In general, $f(x) = \frac{g(x)}{h(x)}$ (rational func) g & h are polynomials

Domain: all #'s excepts those that produce $h(x) = 0$.

Root functions: $f(x) = \sqrt[n]{x}$.

If n is odd, domain is $(-\infty, \infty)$

If n is even, domain is $[0, \infty)$

Ex 3: Consider the function $f(x) = \frac{2x^4 - x^3 + 2x - 1}{4}$.

What type of function is this?

- Polynomial
- Rational
- Root

$$\boxed{f(x) = \frac{2x^4 - x^3 + 2x - 1}{4}} .$$

$$\Rightarrow f(x) = \frac{2}{4}x^4 - \frac{1}{4}x^3 + \frac{2}{4}x - \frac{1}{4}$$

$$= \frac{1}{2}x^4 - \frac{1}{4}x^3 + \frac{1}{2}x - \frac{1}{4}$$

Ex 4: Let $f(x) = 3x^2 + 2x - 5$

a) Evaluate $f(x)$ at 2. Compute $f(2)$.

$$f(2) = 3 \cdot 2^2 + 2 \cdot 2 - 5 = 3 \cdot 4 + 4 - 5 = 12 + 4 - 5 = 11.$$

b) Evaluate $f(x)$ at $x+h$. Compute $f(x+h)$.

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 5 = 3(x^2 + 2xh + h^2) + 2x + 2h - 5$$

$$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 5.$$

c) Simplify $f(x+h) - f(x)$.

$$f(x+h) - f(x) = (3x^2 + 6xh + 3h^2 + 2x + 2h - 5) - (3x^2 + 2x - 5)$$

$$= \cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h \cancel{-5} \cancel{-3x^2} \cancel{-2x} \cancel{+5}$$

$$= 3h^2 + 6xh + 2h.$$

d) Simplify $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{3h^2 + 6xh + 2h}{h} = \frac{\cancel{h}(3h + 6x + 2)}{\cancel{h}} = 6x + 3h + 2.$$

Ex 5: Consider $f(x) = 3x^2 - 5$. Compute $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 5 - 3x^2 + 5}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-5} \cancel{-3x^2} \cancel{+5}}{h}$$

$$= \frac{3h^2 + 6xh}{h} = \frac{\cancel{h}(3h + 6x)}{\cancel{h}} = 6x + 3h.$$